

Tangenten an einen Kreis von einem Punkt außerhalb;

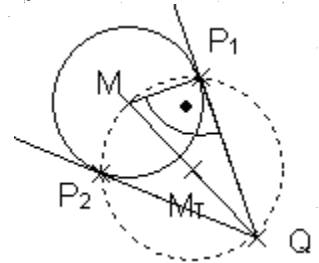
$$k: r = 5 \quad M(2/3) \quad Q(9/2)$$

Sei  $P(p_1/p_2)$  ein Berührungspunkt dann gilt

$$(1) P \in k \quad k: (\vec{x} - \vec{m})^2 = r^2$$

$$I] \left[ \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right]^2 = 5^2$$

$$(2) \vec{MP} \perp \vec{QP} \quad II] \begin{pmatrix} p_1 - 2 \\ p_2 - 3 \end{pmatrix} \cdot \begin{pmatrix} p_1 - 9 \\ p_2 - 2 \end{pmatrix} = 0$$



$$I] (p_1 - 2)^2 + (p_2 - 3)^2 = 25$$

$$II] (p_1 - 2)(p_1 - 9) + (p_2 - 3)(p_2 - 2) = 0$$

$$I] p_1^2 + p_2^2 - 4p_1 - 6p_2 = 12$$

$$II] p_1^2 + p_2^2 - 11p_1 - 5p_2 = -24 \quad \leftarrow \ominus \text{ "II - I"}$$

$$I] p_1^2 + p_2^2 - 4p_1 - 6p_2 = 12$$

$$II - I \rightarrow III] -7p_1 + 1p_2 = -36 \Rightarrow p_2 = 7p_1 - 36 \quad [IV]$$

$$I] p_1^2 + p_2^2 - 4p_1 - 6p_2 = 12$$

$$IV] p_2 = 7p_1 - 36 \quad \leftarrow \text{Einsetzen}$$

$$IV \text{ in } I: p_1^2 + (7p_1 - 36)^2 - 4p_1 - 6(7p_1 - 36) = 12$$

$$\Leftrightarrow 50p_1^2 - 550p_1 + 1500 = 0 \quad | :50$$

$$\Leftrightarrow p_1^2 - 11p_1 + 30 = 0$$

$$\Leftrightarrow p_{1,1} = 6 \quad \vee \quad p_{1,2} = 5 \quad \left. \begin{array}{l} p_1(6/6) \\ p_2(6/-1) \end{array} \right\}$$

$$\text{aus IV} \Rightarrow p_{2,1} = 6 \quad p_{2,2} = -1 \quad \left. \begin{array}{l} p_1(6/6) \\ p_2(6/-1) \end{array} \right\}$$

Tangentengleichungen: Richtvekt =  $\vec{PQ}$ :  $\vec{P_1Q} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ ;  $\vec{P_2Q} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$$t_1: \vec{x} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} + s_1 \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad t_2: \vec{x} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} + s_2 \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$