Nr 2 $\quad P(2,2,1)$, $P_1(3,4,-2)$, $P_2(1{,}5,2,0)$

$$g_{P_1P_2}: \ \vec{x} = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \lambda \cdot \overrightarrow{P_1P_2} = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \lambda \begin{pmatrix}1.5-3\\2-4\\0+2\end{pmatrix}$$

$$\vec{x} = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \lambda \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix}$$

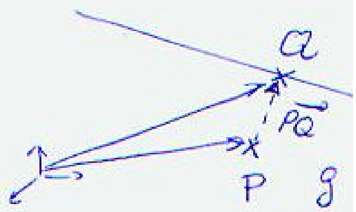
2a) $P \notin g$?

$$\begin{pmatrix}2\\2\\1\end{pmatrix} = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \lambda \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix} \Rightarrow \begin{matrix}-1 = 1{,}5\lambda & \Rightarrow & \lambda = -\frac{2}{3}\\ -2 = -2\lambda & \Rightarrow & \lambda = 1\\ 3 = 2\lambda & \Rightarrow & \lambda = 3/2\end{matrix}$$

$$\Rightarrow P \notin g$$

2b) Gesucht ist der Lotfußpunkt Q; der Abstand d ist dann $d = |\overrightarrow{PQ}|$.

Planfigur:



Es gilt: (1) $Q = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \lambda \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix}$

(2) $\overrightarrow{PQ} = Q - P = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \lambda \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix} - \begin{pmatrix}2\\2\\1\end{pmatrix} = \begin{pmatrix}1\\2\\-3\end{pmatrix} + \lambda \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix}$

(3) $\overrightarrow{PQ} \circ \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix} = 0$ (Skalarprodukt ist Null, da $\overrightarrow{PQ} \perp g$)

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(2) in (3) einsetzen zur Berechnung von λ f. Q

$$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix} + \lambda \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix}\right) \circ \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix} = 0$$

$$(1{,}5 - 4 - 6) + \lambda(2{,}25 + 4 + 4) = 0$$

$$-8{,}5 + 10{,}25\lambda = 0$$

$$\lambda = + \frac{\cancel{17}^{\,1}}{\cancel{2}} \cdot \frac{\cancel{4}^{\,2}}{\cancel{41}_{\,3}} = \frac{2}{3}$$

$$Q = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \frac{2}{3} \cdot \begin{pmatrix}1{,}5\\-2\\2\end{pmatrix} = \begin{pmatrix}4\\8/3\\-2/3\end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix}4-2\\8/3-2\\-2/3-1\end{pmatrix} = \begin{pmatrix}2\\2/3\\-5/3\end{pmatrix}$$

$$|\overrightarrow{PQ}| = \tfrac{1}{3} \cdot \sqrt{65}$$

2c) Schnitt mit $E_{x_{1-2}}$ $\Rightarrow x_3 = 0$

$$\begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1,5 \\ -2 \\ 2 \end{pmatrix}$$

$x_1 - 1,5\lambda = 3$ $\qquad x_1 = 3 + 1,5 = 4,5$

$x_2 + 2\lambda = 4$ $\Rightarrow x_2 = 4 - 2 = 2$

$0 - 2\lambda = -2$ $\Rightarrow \lambda = 1$

$$S_{x_{1-2}} = \begin{pmatrix} 4,5 \\ 2 \\ 0 \end{pmatrix}$$

Schnitt mit $E_{x_{1-3}}$ $\Rightarrow x_2 = 0$

$$\begin{pmatrix} x_1 \\ 0 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1,5 \\ -2 \\ 2 \end{pmatrix}$$

$$\left.\begin{array}{lll} x_1 - 1,5\lambda = 3 & \Rightarrow & x_1 = 6 \\ 0 + 2\lambda = 4 & \Rightarrow & \lambda = 2 \\ x_3 - 2\lambda = -2 & \Rightarrow & x_3 = 2 \end{array}\right\} S_{x_{1-3}} = \begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix}$$

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